

Euler cauchy eqⁿ: has the form

$$x^n \frac{d^n y}{dx^n} + a_0 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = Q(x)$$

$a_0, a_1, \dots, a_n$ are constant, $Q(x)$ is a fⁿ. of x .

Method: 1) Put $x = e^z$

2) Replace $x \frac{dy}{dx} = Dy$ $D = \frac{d}{dz}$

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$$

$$x^n \frac{d^n y}{dx^n} = D(D-1)(D-2) \dots (D-n+1)y$$

$\underbrace{\hspace{10em}}_{\{D-(n-1)\}}$

3) $z = \log x$

Q.1) $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin \log x$

Substituting $x = e^z$, D.E. transforms to.

$$\{D(D-1) - 3D + 5\}y = e^{2z} \sin z \quad D = \frac{d}{dz}$$

$$\{D^2 - D - 3D + 5\}y = e^{2z} \sin z$$

$$(D^2 - 4D + 5)y = e^{2z} \sin z$$

A.E. $m^2 - 4m + 5 = 0$

$$m = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$y_c = e^{2z} \{C_1 \cos z + C_2 \sin z\} = C_1 e^{2z} \cos z + C_2 e^{2z} \sin z$$

$$y_p = \frac{1}{F(D)} V(z) = \frac{1}{D^2 - 4D + 5} e^{2z} \sin z$$

$$= e^{2z} \frac{1}{(D+2)^2 - 4(D+2) + 5} \sin z$$

$$= e^{2z} \frac{1}{D^2 + 4D + 4 - 4D - 8 + 5} \sin z$$

$$= e^{2z} \frac{1}{D^2 + 1} \sin z = e^{2z} \cdot -\frac{z}{2} \cos z$$

$$= -\frac{z}{2} e^{2z} \cos z$$

$$y = C \cdot F + P \cdot I.$$

$$= e^{2z} (C_1 \cos z + C_2 \sin z) - \frac{z}{2} e^{2z} \cos z$$

$$= x^2 (C_1 \cos \log x + C_2 \sin \log x) - \frac{\log x \cdot x^2}{2} \cos \log x.$$

$$Q. \quad x^3 \frac{d^3 y}{dx^3} - 4x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} - 8y = 0$$

$$\{D(D-1)(D-2) - 4D(D-1) + 8D - 8\} y = 0 \quad D = \frac{d}{dx}$$

$$(D^2 - 7D + 14D - 8) y = 0$$

$$A.O.E. \quad m^2 - 7m + 14m - 8 = 0$$

$$(m-1)(m^2 - 6m + 8) = 0$$

$$(m-1)(m-2)(m-4) = 0$$

$$m = 1, 2, 4$$

$$y_c = C_1 e^x + C_2 e^{2x} + C_3 e^{4x}$$

$$= C_1 e^x + C_2 (e^x)^2 + C_3 (e^x)^4$$

$$= C_1 x + C_2 x^2 + C_3 x^4 \quad \underline{\text{Ans.}}$$